Stochastic Rendering of Density Fields

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Stochastic Models

Phenomenon modelled as a random field $\rho(\mathbf{x}, t)$

Random field is determined by its second-order statistics:

- Mean $\mu(\mathbf{x}, t) = \langle \rho(\mathbf{x}, t) \rangle$
- Structure (covariance)
  $C_\rho(\mathbf{x}, t; \mathbf{x}', t') = \langle \rho(\mathbf{x}, t) \rho(\mathbf{x}', t') \rangle - \mu(\mathbf{x}, t) \mu(\mathbf{x}', t')$

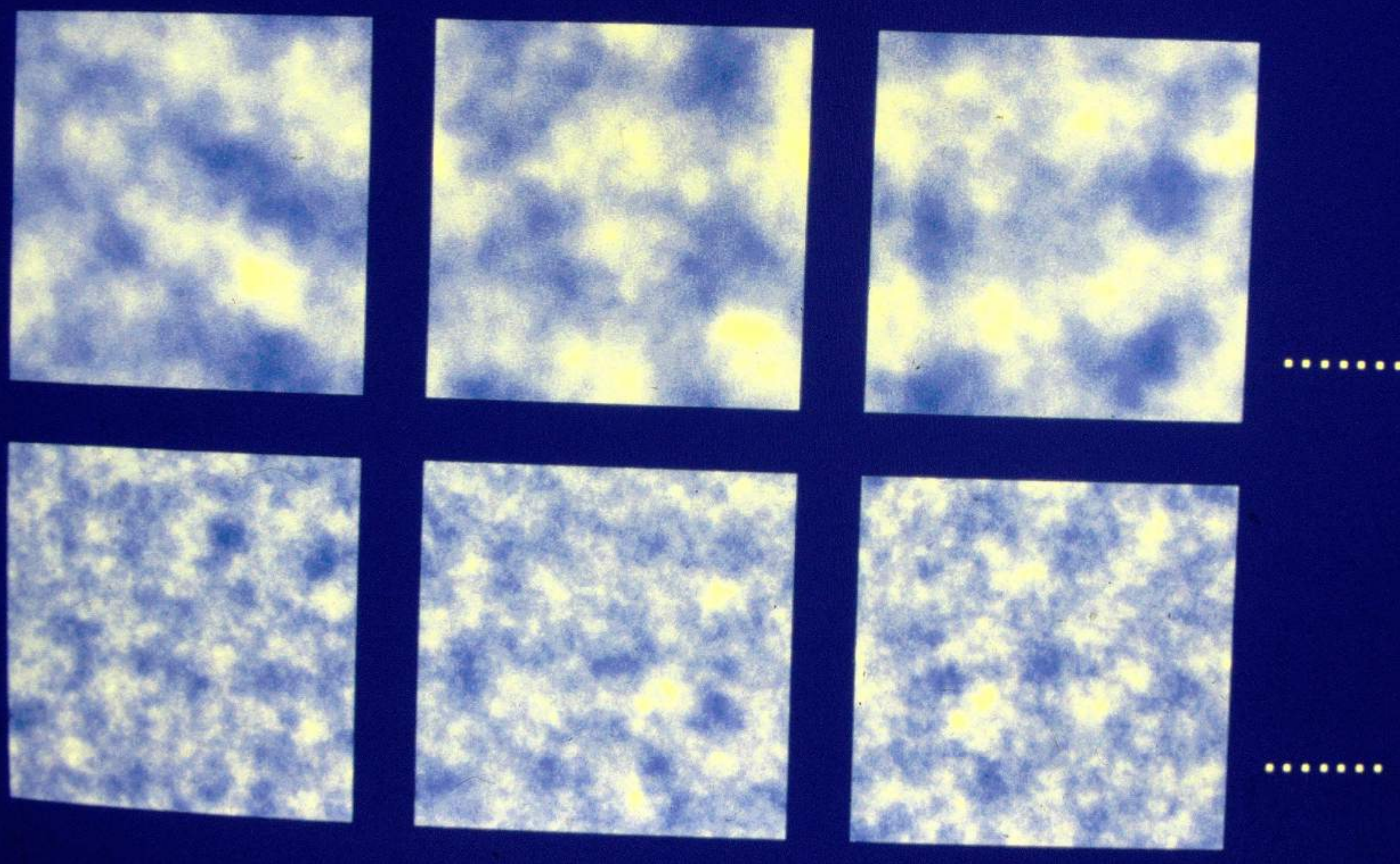
Homogeneous random fields satisfy

$$\mu(\mathbf{x}, t) = \text{constant}$$

$$C_\rho(\mathbf{x}, t; \mathbf{x}', t') = C_\rho(\mathbf{x}' - \mathbf{x}, t' - t)$$

Realization of a Random Field

Statistics do not define a single function



Rendering of Stochastic Models

Traditionally:

- Generate realization of stochastic model
- Render realization

Disadvantages:

- Rendering computationally expensive
- Sampling problem

Stochastic Rendering

Model intensity field as a random function

- Calculate statistics of intensity field from
  - * Statistics of the stochastic model
  - * Illumination equation
- Generate a realization of the intensity field

Analogy with texture mapping

Examples of Stochastic Rendering

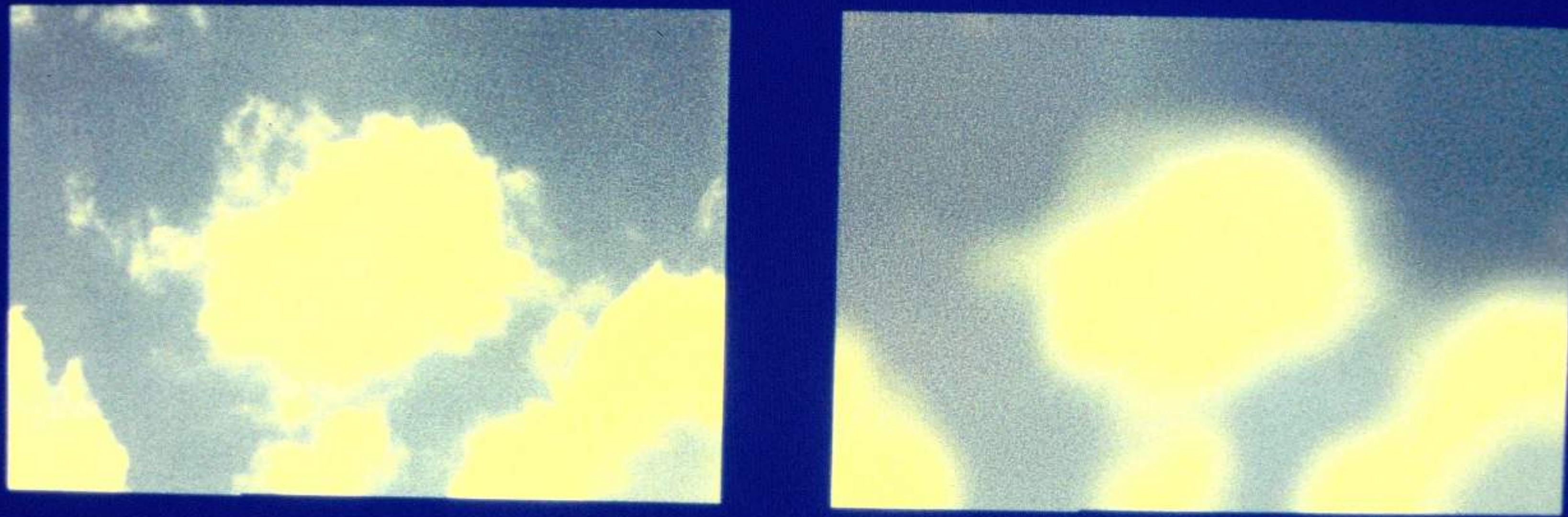
Surface reflectance models

- Mean value Torrance and Sparrow 67 & He et al 91
- Correlation Krueger 88

Rendering of clouds Gardner 85

- Illumination equation not physics-based
- No true volume rendering (surface-based)

Stochastic Model for Density Fields



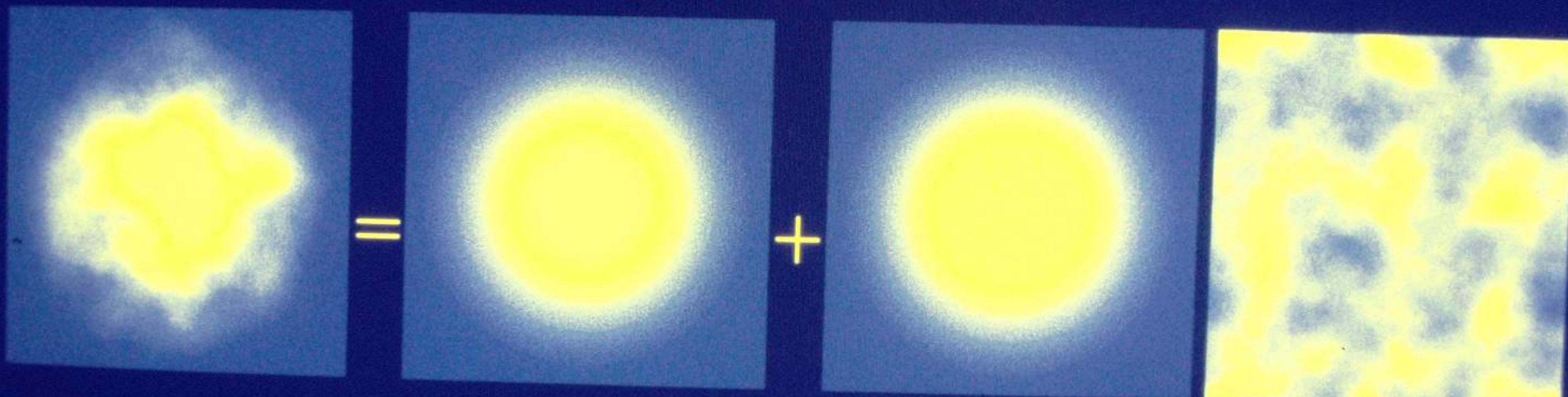
Observations:

- Mean value is "blobby-like"
- Variance depends on the mean

→ cannot use homogeneous random fields

Transformation of a Homogeneous Field

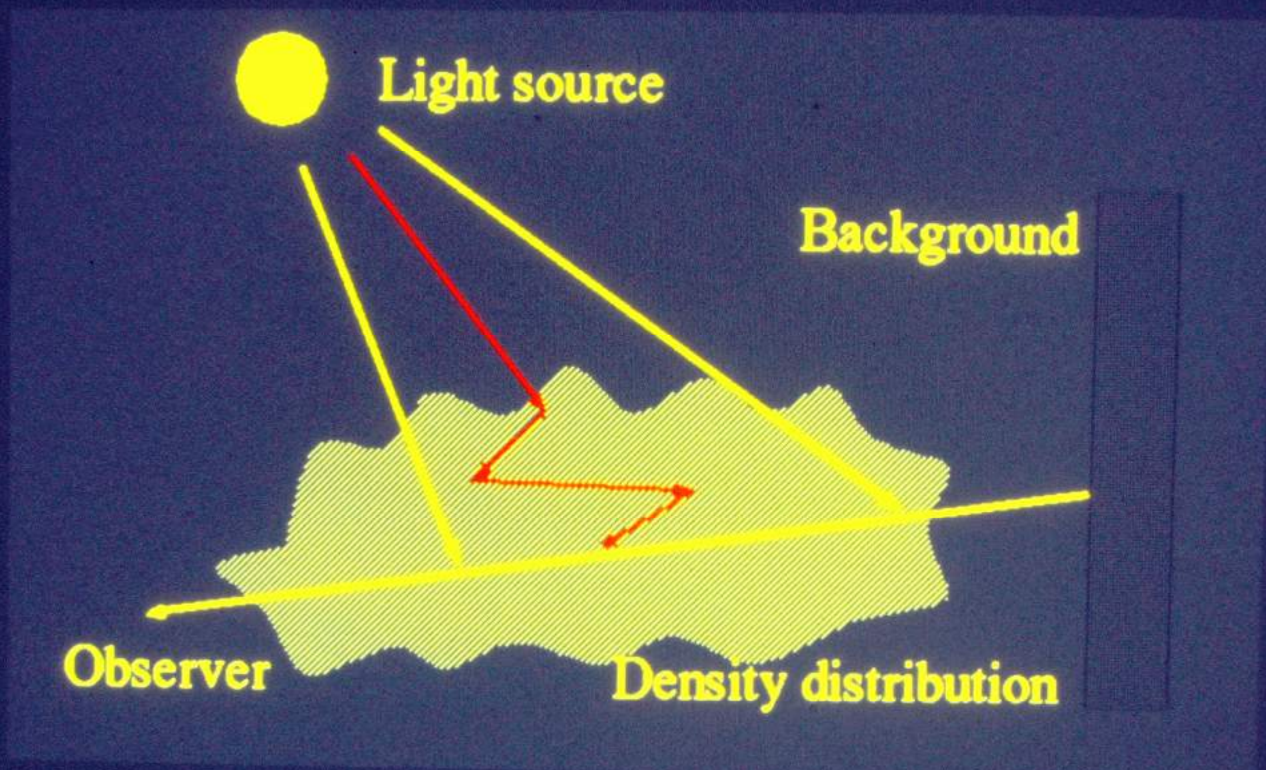
$$\rho(\mathbf{x},t) = f(\mathbf{x},t) + g(\mathbf{x},t)q(\mathbf{x},t)$$



Statistics of the transformation are known $\langle q \rangle = 0$

- mean $= f(\mathbf{x},t)$
- covariance $= C_q(\mathbf{x}' - \mathbf{x}, t' - t)g(\mathbf{x},t)g(\mathbf{x}',t')$

Illumination Model



$$I_{observer} = \tau\ I_{background} + (1-\tau) I_{gas}$$

Transparency

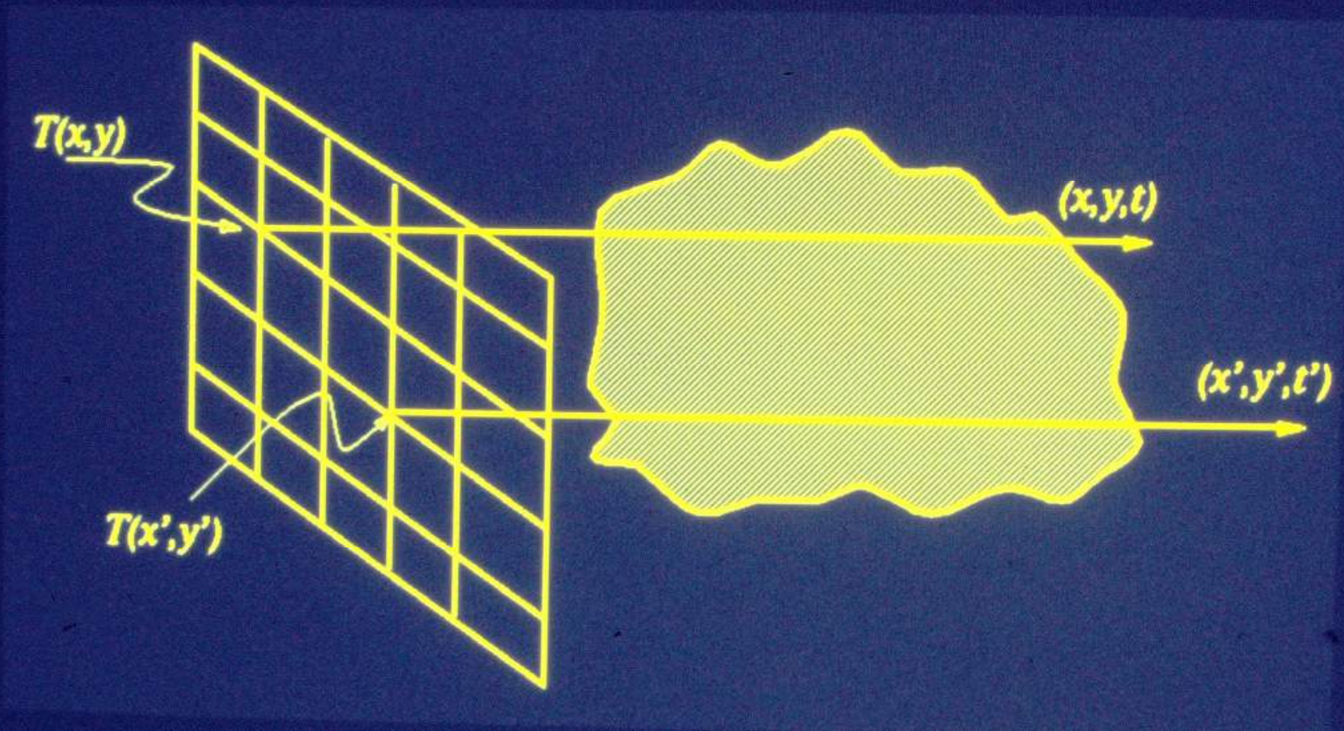
Illumination due to the gas is smooth and can be calculated using mean density

→ only the transparency is random
→ determine statistics of

$$\tau(0,b) = \exp\left(-\int_0^b \rho(\mathbf{x}_0 + uD)\, du\right)$$

Statistical Equations

Assume orthographic projection



Determine mean and covariance of the function

$$T(x,y) = \int_0^b \rho(x,y,u)\,du$$

Statistics of $T(x, y)$

- mean $F(x, y) = \int_0^b f(x, y, u)\, dt$
- variance $G(x, y) = \int_0^b g(x, y, u)\, du$
- covariance $= C_q(x' - x, y' - y, 0)G(x, y)G(x', y')$

$$\Longrightarrow T(x, y) = F(x, y) + G(x, y)Q(x, y)$$

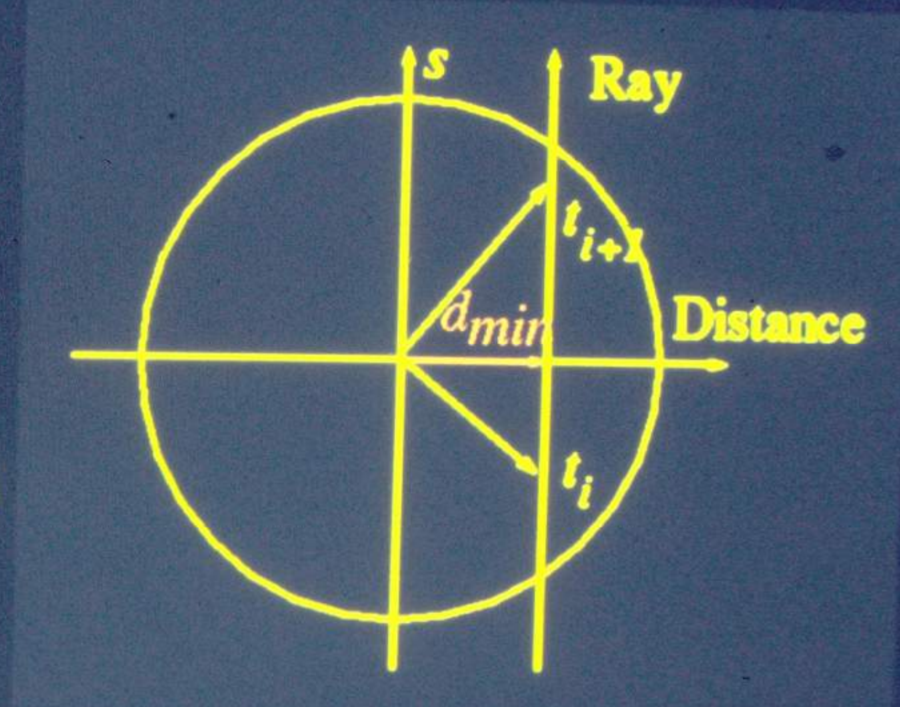
where Q can be any "slice" of the field $q(x, y, z^*)$

Compare with

$$\rho(x, y, z) = f(x, y, z) + g(x, y, z)q(x, y, z)$$

Computation of $T(x, y)$

Mean F and variance G are both integrals of the blobby fields f and g
→ efficient computation
see Stam & Fiume 93



Compute $Q(x, y)$ by sampling $q(x, y, z^*)$ consistently
Possible choice: $z^* =$ point nearest to blob on the ray

→ image and frame to frame coherence

Overview of Rendering Algorithm

$T = 0$
for each blob intersecting ray do
    Calculate F and G
    Calculate Q by sampling q at midpoint
    Update integral: $T = T + F + GQ$
end for

$I_{background}$ obtained from standard ray-tracer
Calculate transparency: $\tau = \exp(-T)$
Calculate I_{gas} from mean density f only
Combine: $I = \tau I_{background} + (1 - \tau) I_{gas}$

Results

Clouds

- Single cloud
- Group of clouds
- Interaction with objects (CN Tower)
- Frame to frame coherence (Fly through)

Fire