

An Illumination Model for a Skin Layer Bounded by Rough Surfaces

Jos Stam

Alias | wavefront

Seattle, USA

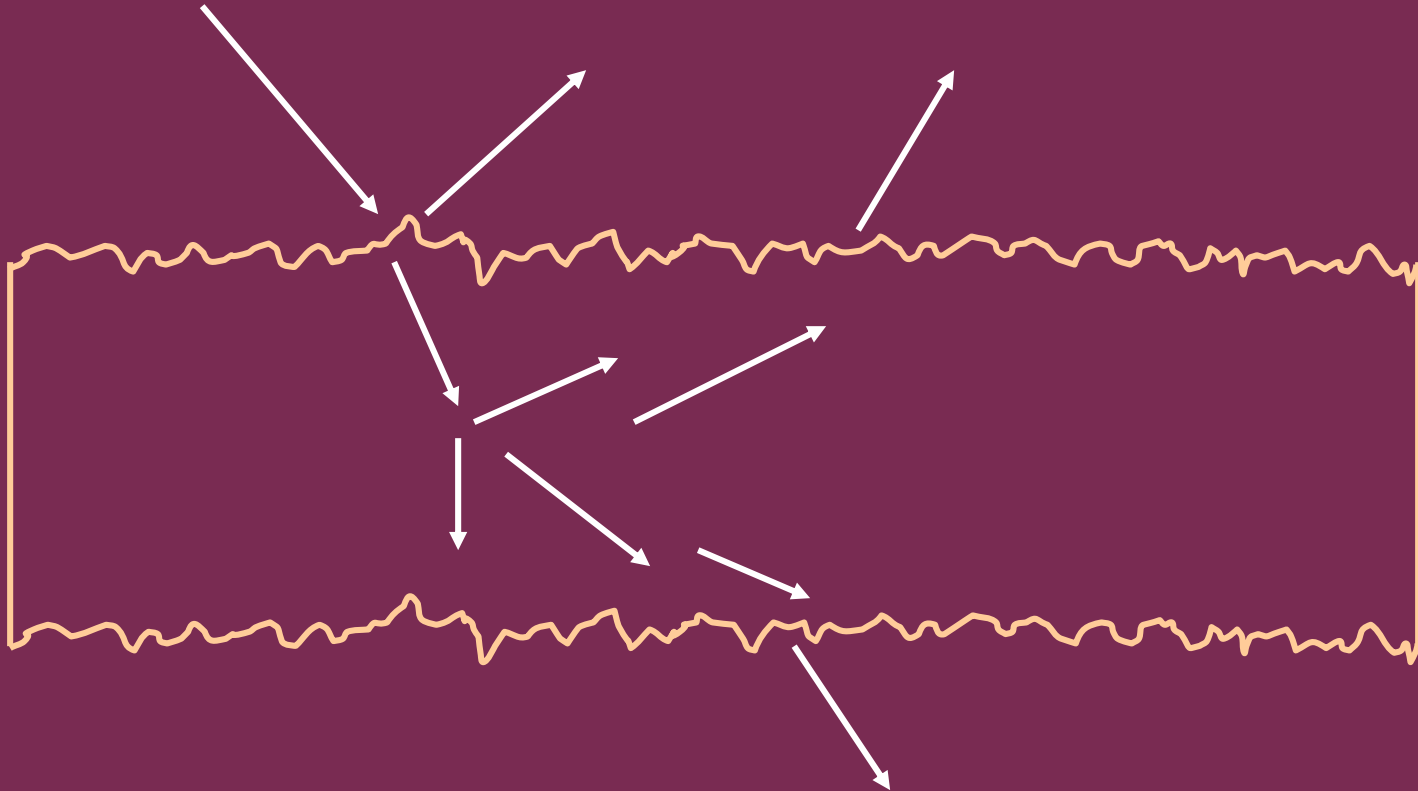
Motivation

Render effects of subsurface scattering
(non-metals)

Two important examples:

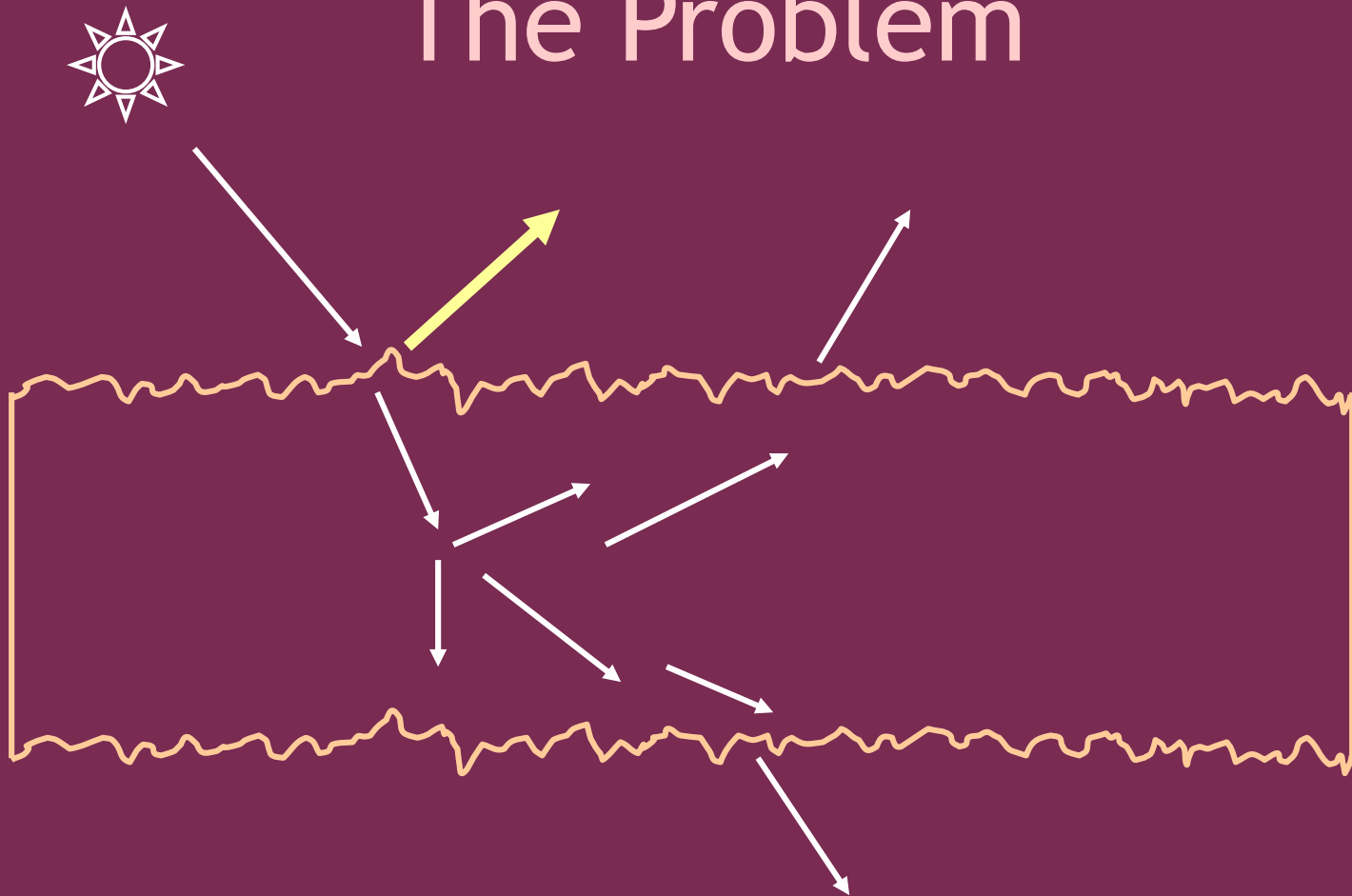
- Skin (Entertainment Industry)
- Paint (Automotive Industry)

The Problem



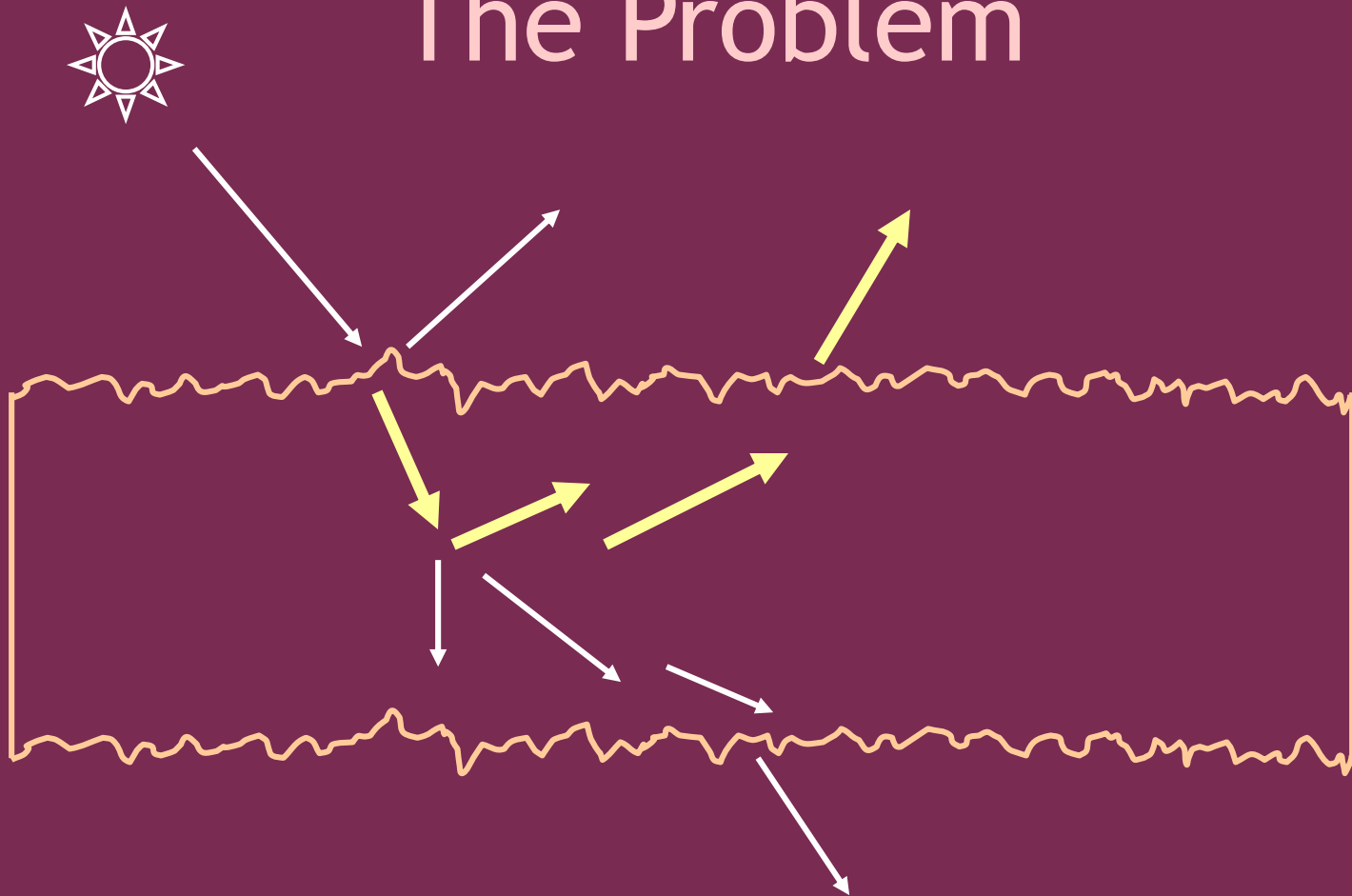
| = ...

The Problem



$$I = I_{\text{surface}} + \dots$$

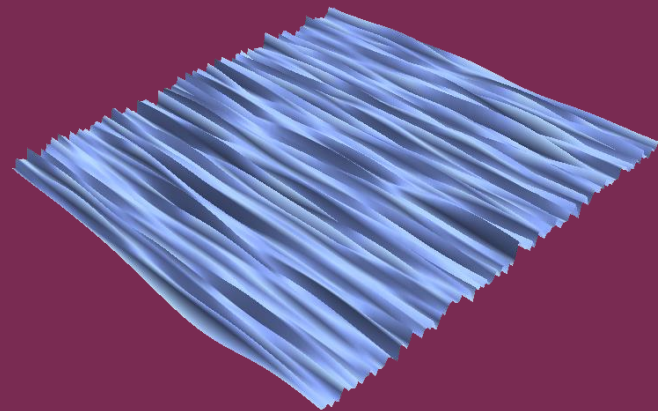
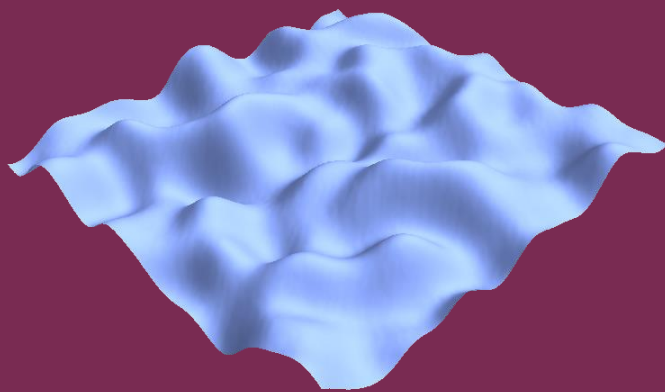
The Problem



$$I = I_{\text{surface}} + I_{\text{subsurface}}$$

Surface Reflection & Refraction

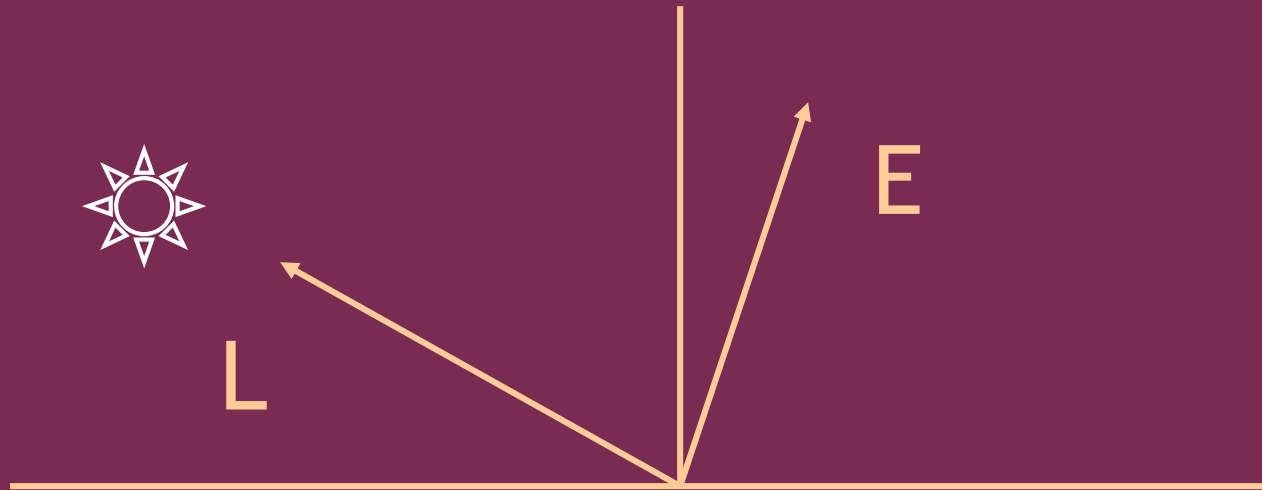
Use random surface height model



Cook-Torrance 81 (Beckmann)

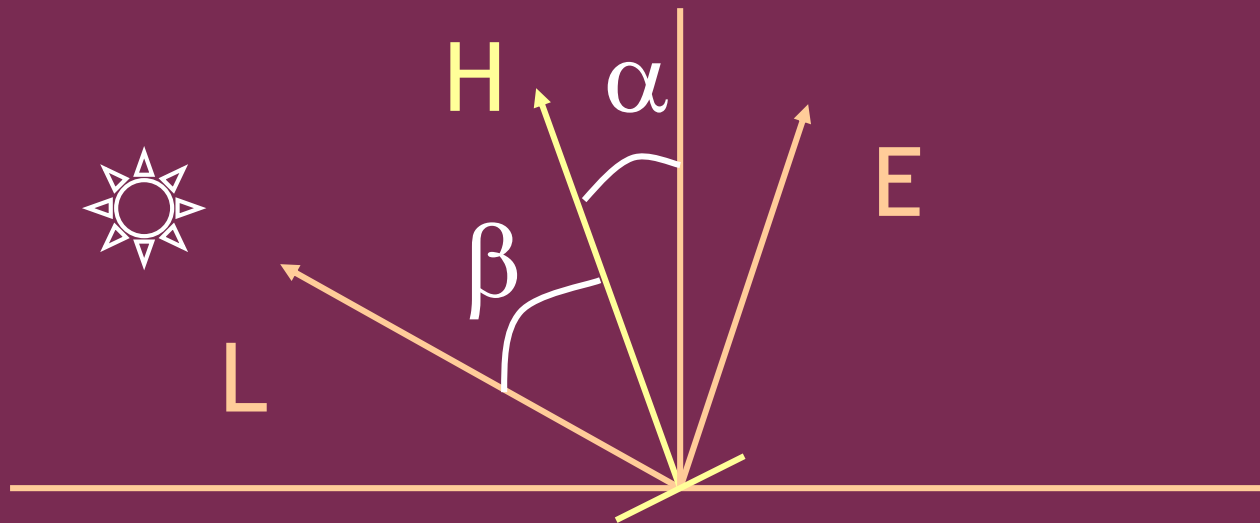
He-Torrance 91 + Stam 99 (Wave Effects)

Reflection



Given L and E, find H

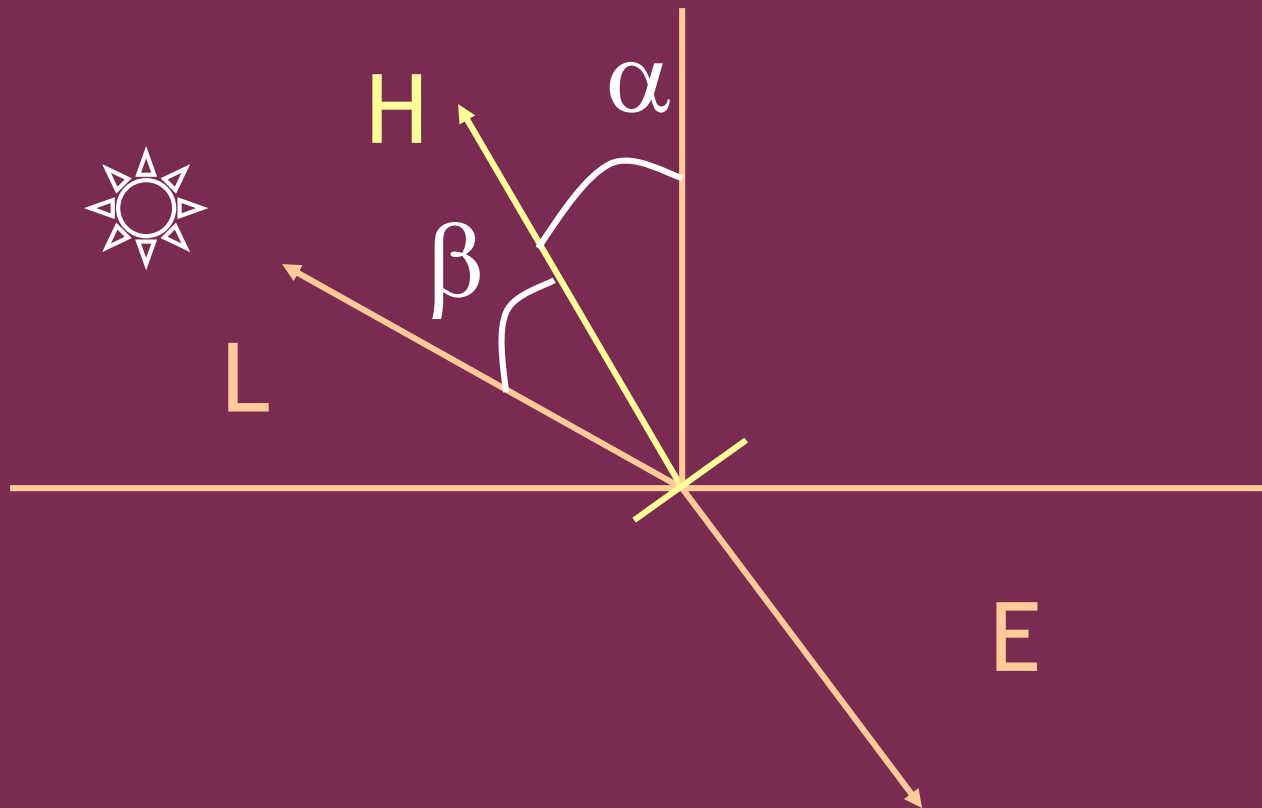
Reflection



Given L and E , find H

$$\text{BRDF} \sim \text{Fresnel}(\beta, \eta) \text{ Beck}(\alpha, \sigma)$$

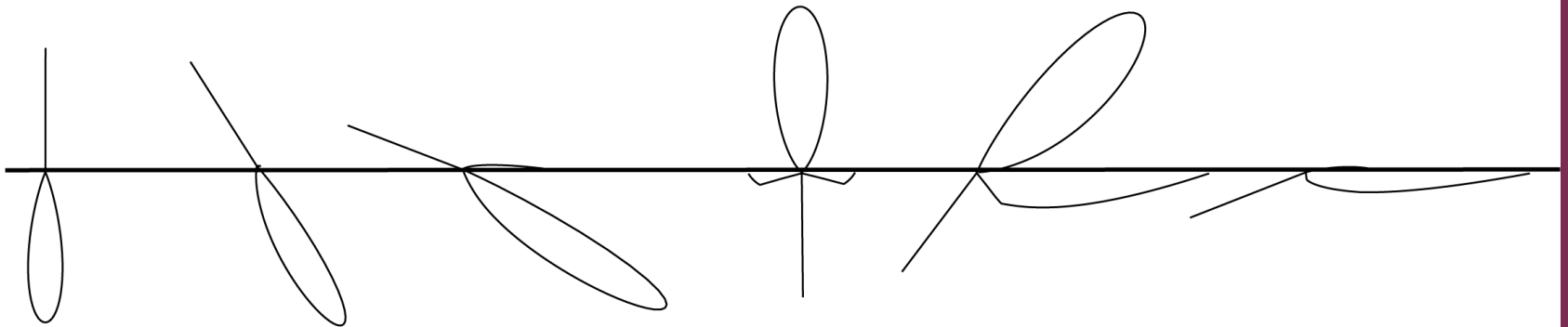
Refraction



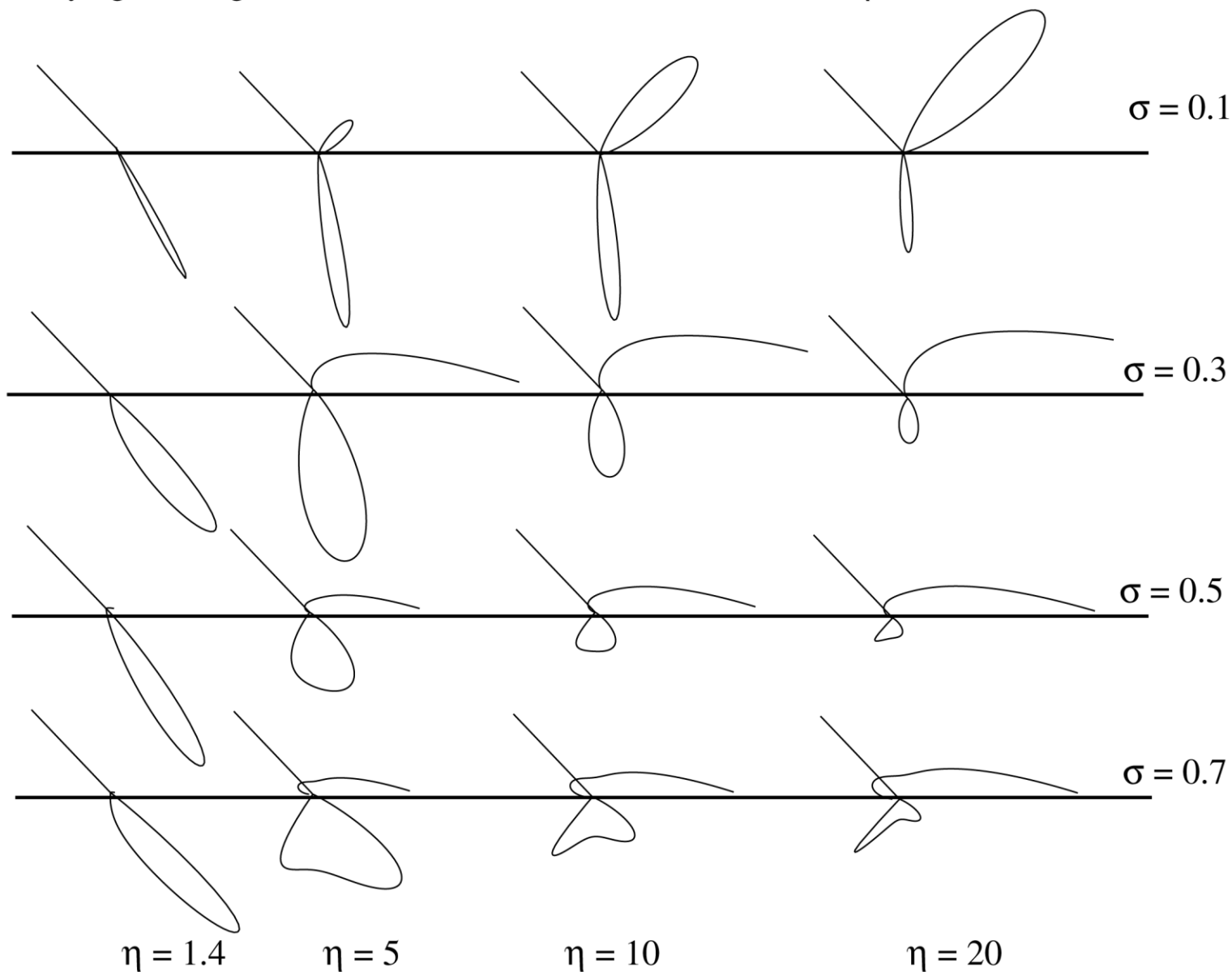
$$\text{BTDF} \sim [1 - \text{Fresnel}(\beta, \eta)] \text{Beck}(\alpha, \sigma)$$

Results

BRDF and BDTF at the skin's surface $\sigma=0.5$ $\eta=1.4$



Varying the roughness σ and the ratio of indices of refraction η



Subsurface Reflection

Previous work:

Analytical: Lambert ??,
Hanrahan-Krueger 93

Monte-Carlo: Westin et al. 92,
Hanrahan-Krueger 93

BSSRDF: Pharr-Hanrahan 00,
Jensen et al. 01

Experiments

Monte Carlo too expensive

Analytical solutions unavailable

(show demo)

Our Solution

Semi-Analytical:

Solve discrete problem exactly
And fit splines to results

Setting

Homogeneous Layer bounded by rough surfaces

air



air / bone

Parameters of the Skin

surface { Surface roughness : σ
Index of refraction : $\eta \sim 1.4$

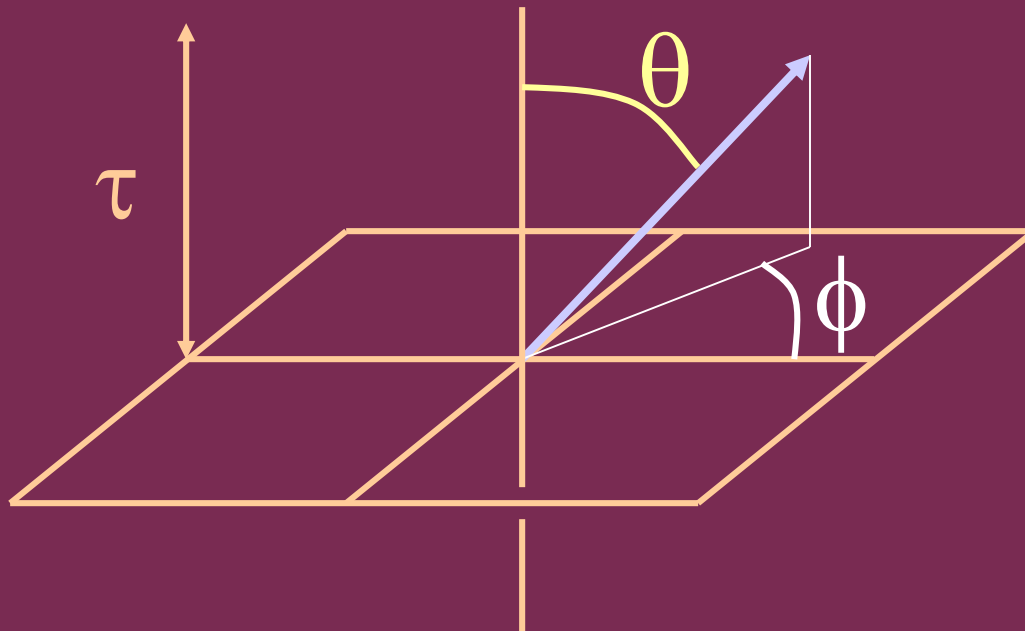
skin { Transparency : $T = \exp(-\tau_b)$
 τ_b : optical depth
Albedo : Ω (amount of scattering)
Anisotropy : g (forward scattering)

Human skin: $\Omega \sim 1$, $g \sim 1$, $T \sim 0$

Discretization

For a given incoming direction (θ_0, ϕ_0)

Intensity : $u(\tau, \theta, \phi)$



Discretization

For a given incoming direction (θ_0, ϕ_0)

Intensity : $u(\tau, \theta, \phi)$

Plan:

- Fourier Transform in ϕ
- Discretize angle θ
- Solve discrete problem

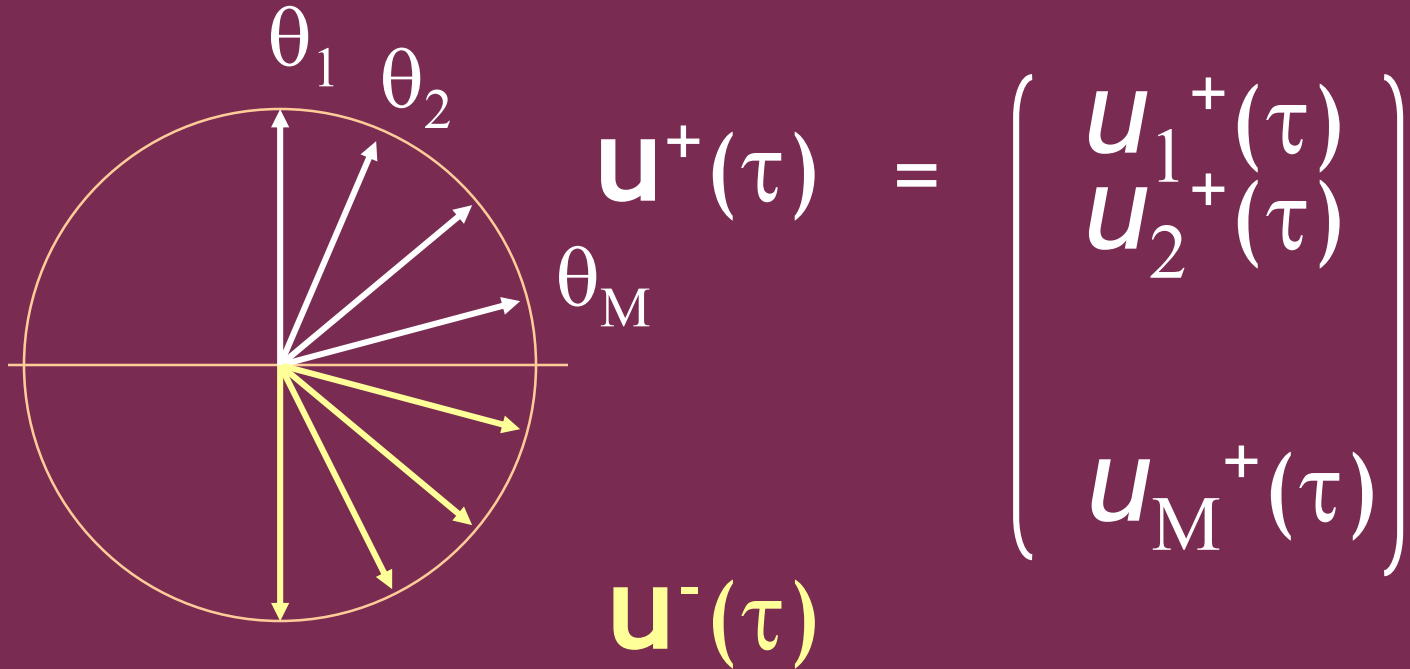
Fourier Transform in ϕ

$$u(\tau, \theta, \phi) = \sum_{k=0}^N u_k(\tau, \theta) \cos(2\pi k\phi)$$

(N+1) Independent Equations

-> only focus on one term

Discrete Ordinates for θ



Equation of Transfer

$$\underbrace{\frac{d}{d\tau} \begin{pmatrix} \mathbf{u}^+ \\ \mathbf{u}^- \end{pmatrix}}_{\text{Change}} = \underbrace{- \begin{pmatrix} \mathbf{u}^+ \\ \mathbf{u}^- \end{pmatrix}}_{\text{out-scatter} \\ + \text{absorption}} + \underbrace{\begin{pmatrix} S^{++} & S^{+-} \\ S^{-+} & S^{--} \end{pmatrix} \begin{pmatrix} \mathbf{u}^+ \\ \mathbf{u}^- \end{pmatrix}}_{\text{in-scatter}}$$

Equation of Transfer

$$\frac{d}{d\tau} \begin{pmatrix} \mathbf{u}^+ \\ \mathbf{u}^- \end{pmatrix} = \begin{pmatrix} \mathbf{M}^{++} & \mathbf{M}^{+-} \\ \mathbf{M}^{-+} & \mathbf{M}^{--} \end{pmatrix} \begin{pmatrix} \mathbf{u}^+ \\ \mathbf{u}^- \end{pmatrix}$$

1D equivalent:

$$\dot{x} = a x \quad x(t) = \exp(at)x_0$$

Diagonalize

$$\frac{d}{d\tau} \begin{pmatrix} \mathbf{w}^+ \\ \mathbf{w}^- \end{pmatrix} = \begin{pmatrix} \mathbf{L} & \mathbf{0} \\ \mathbf{0} & -\mathbf{L} \end{pmatrix} \begin{pmatrix} \mathbf{w}^+ \\ \mathbf{w}^- \end{pmatrix}$$

Solution:

$$\mathbf{w}^+(\tau) = \exp(\tau \mathbf{L}) \mathbf{w}^+_0$$

$$\mathbf{w}^-(\tau) = \exp(-\tau \mathbf{L}) \mathbf{w}^-_0$$

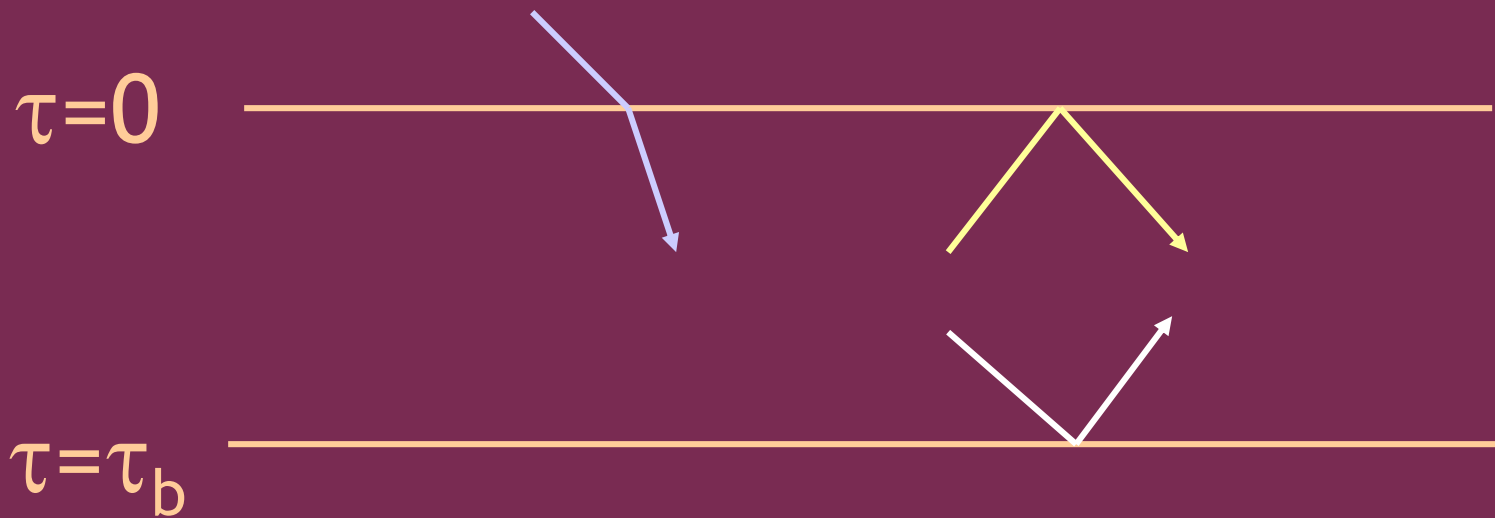
Diagonalize

$$\frac{d}{d\tau} \begin{pmatrix} \mathbf{w}^+ \\ \mathbf{w}^- \end{pmatrix} = \begin{pmatrix} \mathbf{L} & \mathbf{0} \\ \mathbf{0} & -\mathbf{L} \end{pmatrix} \begin{pmatrix} \mathbf{w}^+ \\ \mathbf{w}^- \end{pmatrix}$$

Solution:

$$\begin{aligned} \mathbf{w}^+(\tau) &= \exp(\tau \mathbf{L}) \mathbf{w}_0^+ \\ \mathbf{w}^-(\tau) &= \exp(-\tau \mathbf{L}) \mathbf{w}_0^- \end{aligned} \quad \begin{array}{l} \boxed{\mathbf{w}_0^+} \\ \boxed{\mathbf{w}_0^-} \end{array} \quad \text{unknown}$$

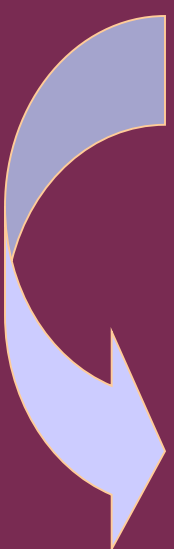
Find Unknowns



$$\mathbf{w}^-(0) = \mathbf{t}_{12} + \mathbf{R}_{21} \mathbf{w}^+(0)$$

$$\mathbf{w}^+(\tau_b) = \mathbf{R}_{23} \mathbf{w}^-(\tau_b)$$

Complete System


$$\begin{cases} \mathbf{w}^+(\tau) = \exp(\tau \mathbf{L}) \mathbf{w}^+_0 \\ \mathbf{w}^-(\tau) = \exp(-\tau \mathbf{L}) \mathbf{w}^-_0 \end{cases}$$

$$\begin{cases} \mathbf{w}^-(0) = \mathbf{t}_{12} + \mathbf{R}_{21} \mathbf{w}^+(0) \\ \mathbf{w}^+(\tau_b) = \mathbf{R}_{23} \mathbf{w}^-(\tau_b) \end{cases}$$

Linear System

$$\begin{cases} \mathbf{w}^+(\tau) = \exp(\tau \mathbf{L}) \mathbf{w}^+_0 \\ \mathbf{w}^-(\tau) = \exp(-\tau \mathbf{L}) \mathbf{w}^-_0 \end{cases}$$

$$\mathbf{w}^-_0 = \mathbf{t}_{12} + \mathbf{R}_{21} \mathbf{w}^+_0$$

$$\exp(\tau_b \mathbf{L}) \mathbf{w}^+_0 = \exp(-\tau_b \mathbf{L}) \mathbf{R}_{23} \mathbf{w}^-_0$$

Linear System

$$\begin{cases} \mathbf{w}^+(\tau) = \exp(\tau \mathbf{L}) \mathbf{w}^+_0 \\ \mathbf{w}^-(\tau) = \exp(-\tau \mathbf{L}) \mathbf{w}^-_0 \end{cases}$$

$$\begin{pmatrix} -\mathbf{R}_{21} & \mathbf{I} \\ \mathbf{I} & -\exp(-2\tau_b \mathbf{L}) \mathbf{R}_{23} \end{pmatrix} \begin{pmatrix} \mathbf{w}^+_0 \\ \mathbf{w}^-_0 \end{pmatrix} = \begin{pmatrix} \mathbf{t}_{12} \\ \mathbf{0} \end{pmatrix}$$

Back to Reality

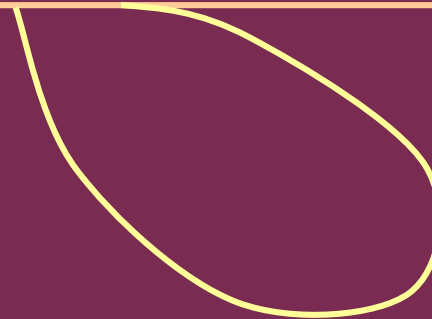
$\tau=0$

$u(0, \theta, \phi)$



$\tau=\tau_b$

$u(\tau_b, \theta, \phi)$



top

$$\mathbf{w}^+(0)$$



$$\mathbf{u}^+(0)$$



$$u_k(0, \theta)$$



$$u(0, \theta, \phi)$$

bottom

$$\mathbf{w}^-(\tau_b)$$



$$\mathbf{u}^-(\tau_b)$$



$$u_k(\tau_b, \theta)$$



$$u(\tau_b, \theta, \phi)$$

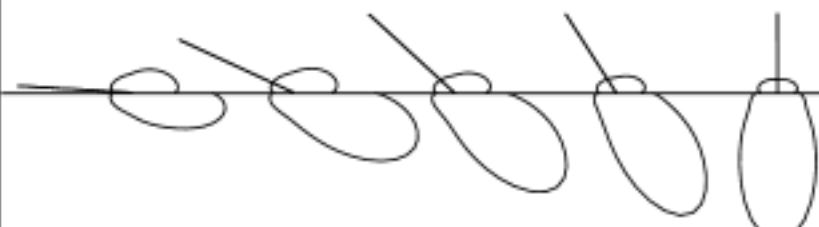
back from eigenspace

fit splines

add Fourier terms

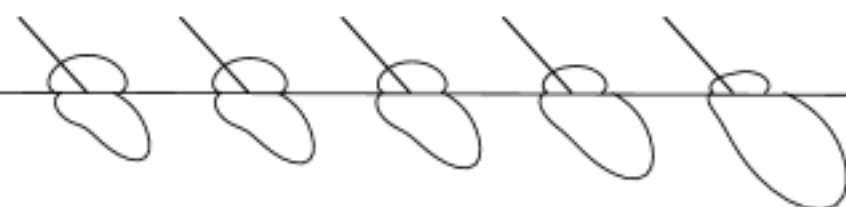
Results

Varying the incident angle



$\theta_0 =$ 85 60 45 30 0
 $g=0.8$ $T=0.2$ $\Omega=1.0$ $\sigma=0.5$

Varying the anisotropy of the layer



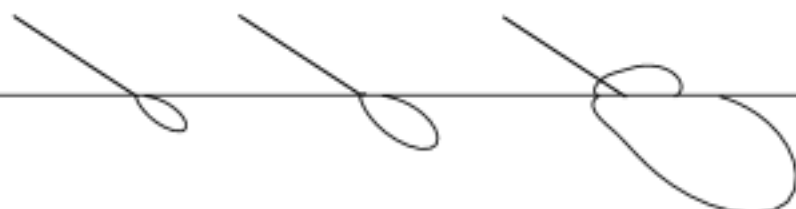
$g =$ 0.0 0.2 0.4 0.6 0.8
 $T=0.2$ $\Omega=1.0$ $\sigma=0.5$

Varying the surface roughness



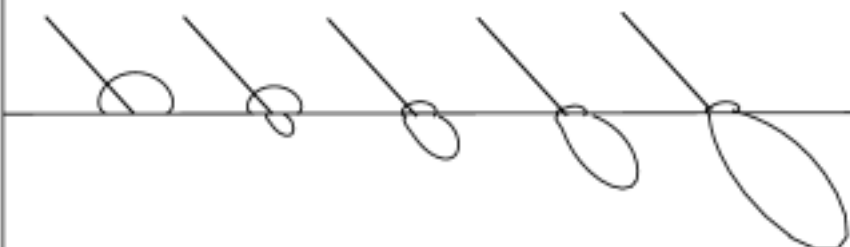
$\sigma =$ 0.1 0.2 0.3 0.5 0.9
 $g=0.8$ $T=0.1$ $\Omega=1.0$

Varying the albedo of the layer



$\Omega =$ 0.1 0.5 1.0
 $g=0.8$ $T=0.2$ $\sigma=0.5$

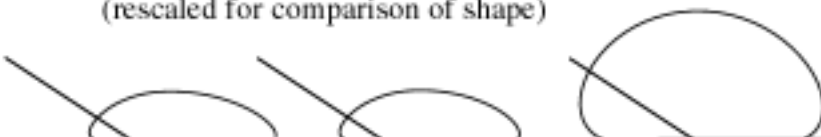
Varying the transparency of the layer



$T =$ 0.0 0.05 0.1 0.3 0.9
 $g=0.8$ $\Omega=1.0$ $\sigma=0.5$

Varying the albedo of the layer

(rescaled for comparison of shape)



$\Omega =$ 0.1 0.5 1.0
 $g=0.8$ $T=0.0$ $\sigma=0.5$

Results

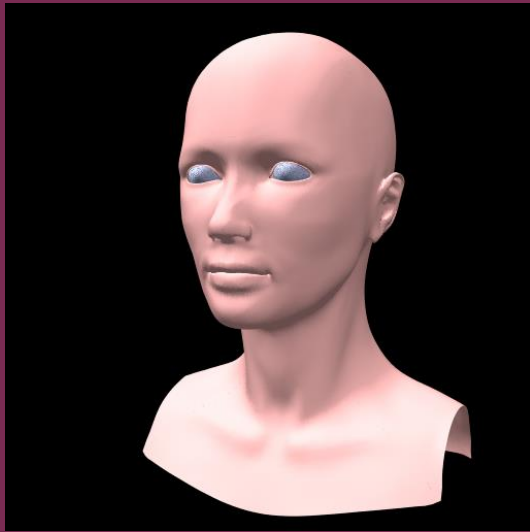
Implemented as a MAYA shader

Available for free at

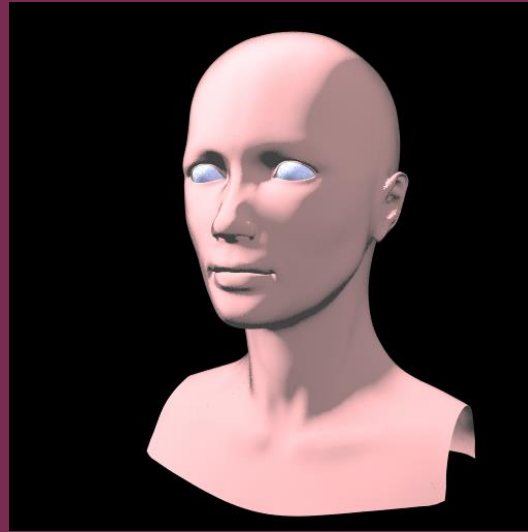
<http://www.aliaswavefront.com>

(follow “community” + “Download”)

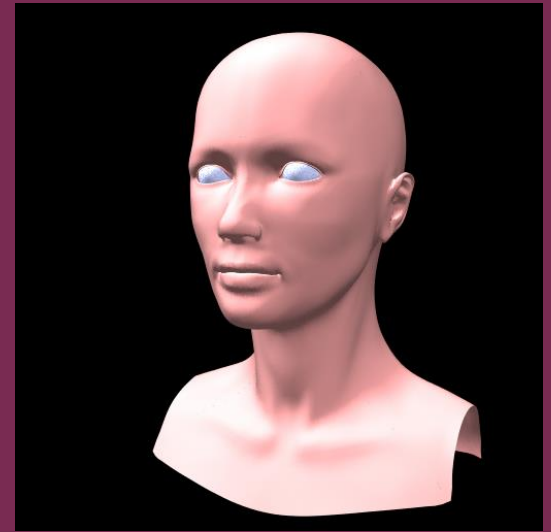
Results



Lambert



Hanrahan-Krueger



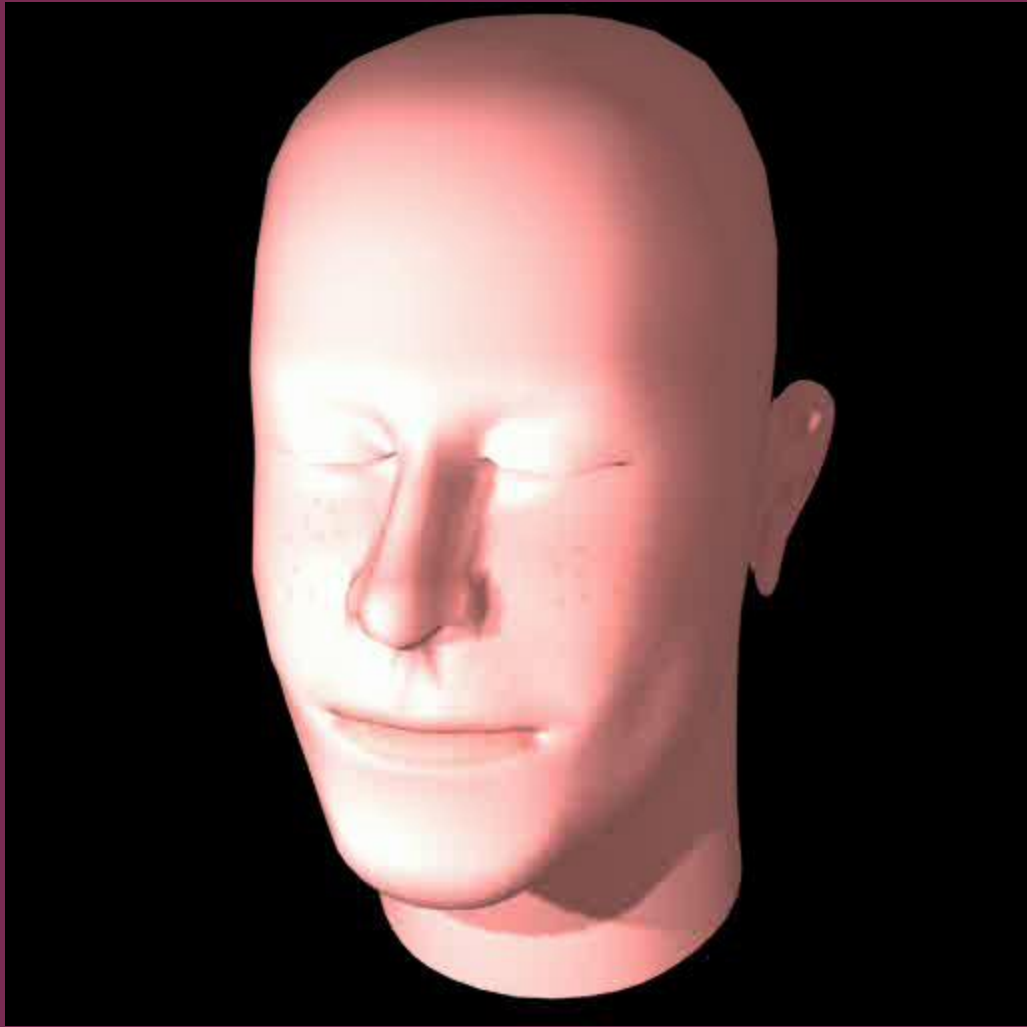
New Model

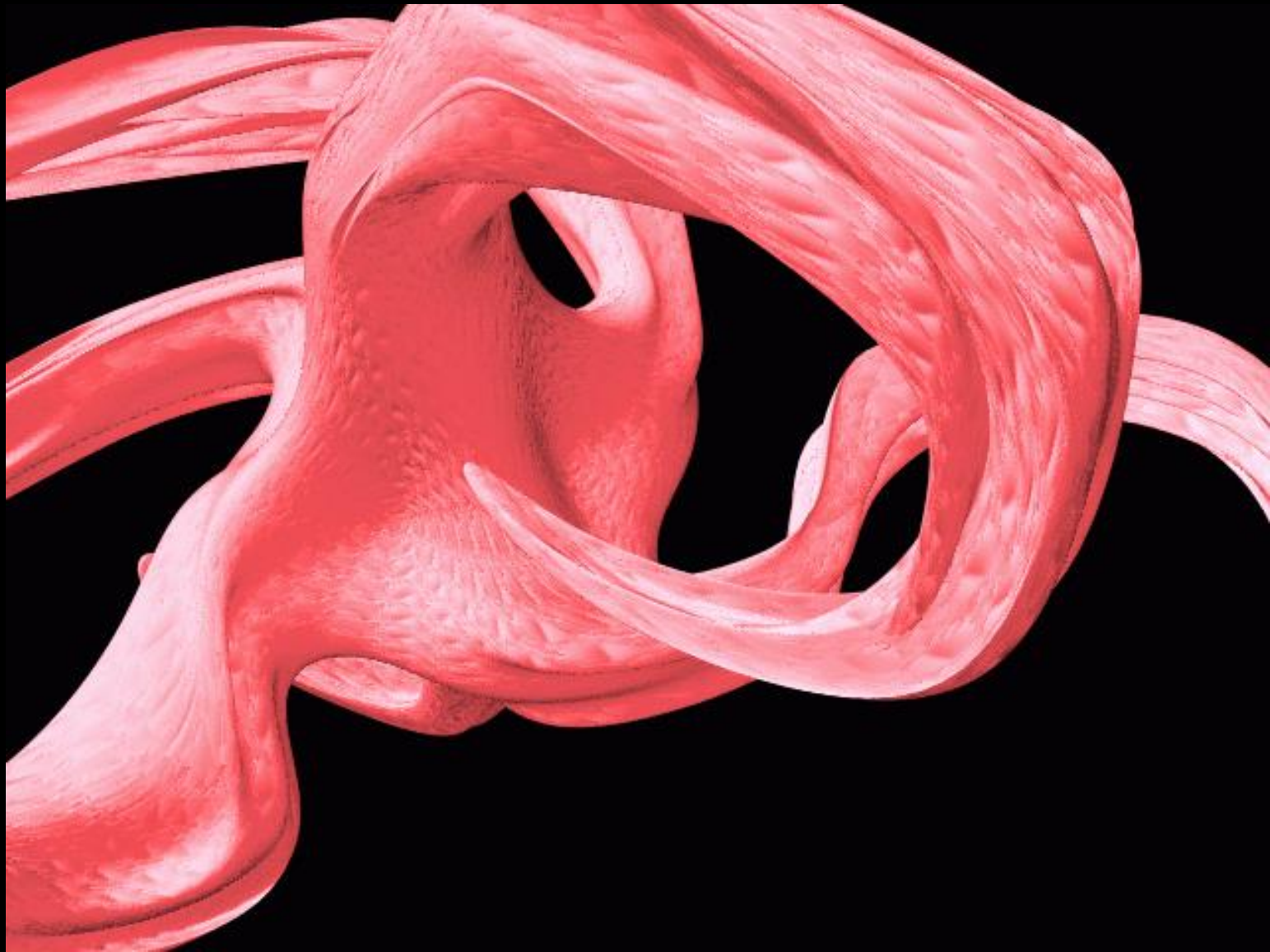


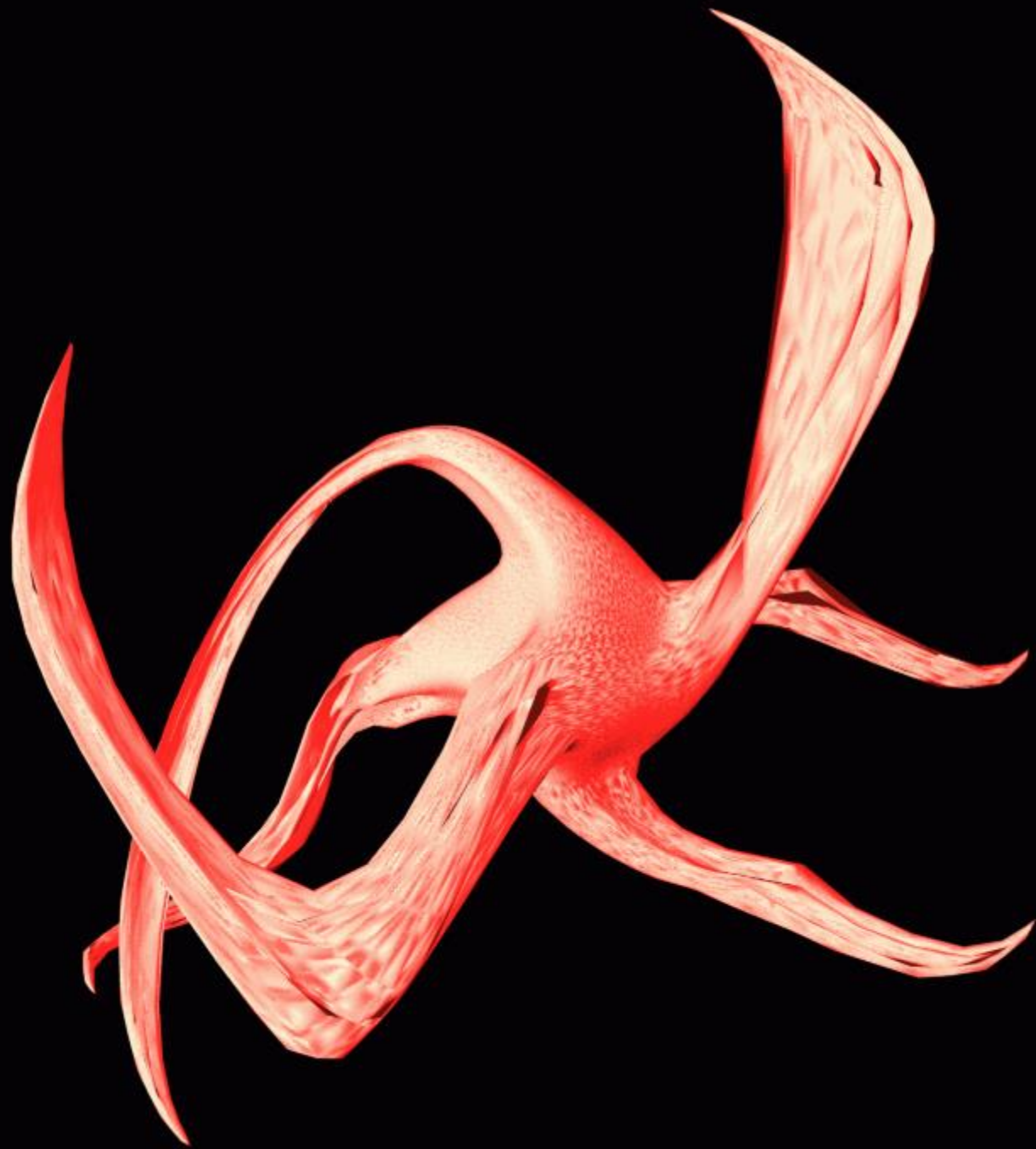












Future Work

- Experimental validation
- More Layers
- Better fits
- Non-homogeneous layer
- BSSRDF